## High-gradient theories as new essential ingredient for the beyond-Standard-Model Des(s)ert cuisine: Getting rid of divergences in Feynman graphs, unified "all-in-one" states and origin of generations

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Inspiring ourselves by the assumption that the notion of symmetry itself is insufficient to construct the consistent physics of the Desert (the so-called region of energies beyond the Standard Model) and some additional insights are needed, we suggests that high-energy theories must take into account the higher-order variations of fields. With this in mind we propose the generalization of the concepts of kinetic energy and free particle. It is shown that the theory founded on such principles reveals major features of the genuinely high-energy one, first of all, it appears to be free from ultra-violet divergences, even the self-energy loop terms become finite. Also we discuss other arising interesting phenomena such as the high-gradient currents and charges, unified "all-in-one" multi-mass states and origin of generations, VR symmetry, regularization-without-renormalization of SM, etc.

PACS number(s): 12.60.-i, 11.10.Cd

Nowadays there is no clear theory describing the fieldparticle habitats of the region of energies between the electroweak and Planck scales. Many physicists even name this region the Desert probably supposing that its nature is rather meager in comparison with its low-energy (Standard Model) and high-energy (superstring) borders. The recently known attempts of generalization of SM are concerned with searches for new groups of symmetries and carriers of appropriate interactions. The dream which is inherent to such searches is the one about the theory which would be genuinely high-energetical in the sense, e.g., free from ultraviolet divergences [1]. For instance, the popularity of supersymmetric theories was arisen principally by their (relatively) good ultraviolet behaviour. The number of proposed since then symmetries and corresponding models is so huge that sometimes it is hard to understand whether a paper is devoted to the group theory or to the high-energy physics.

However, it may happen that the notion of symmetry itself is not enough to understand the physics of the Desert and some additional, rather physical than mathematical, insights are needed. The first insight can appear if we begin to physically think about the nature of the Desert. It is almost doubtless that this region is characterized not only by high values of fields but also by high rates of their variations in space and time. Therefore, the first assertion we can make is: (i) The higher-order field variations can not be neglected in the Desert. Further, let us think about the nature of particles, in particular, about the concept of a free particle (field). Indeed, which entity can be regarded as a free (i.e., non-interacting) particle in the region with non-trivial rapidly varying polarized vacua and hence with large radiation corrections and hence with large self-interaction? How can we define the system of free fields in the high-energy regime? Besides, thinking about the mathematical ways of generalization of the Standard Model why should they be based on the modifying of interaction (potential) terms only? What about the generalization of the kinetic energy itself? In view of the previous assertion we suggest that (ii) The usual definition of the free particle becomes insufficient in the Desert and needs to be modified, as well as that (iii) The notion of the kinetic energy should be generalized to take into account the phenomena which appear in the Desert.

So, let us try to materialize these three statements in local Lagrangians. Let us begin with the generic real scalar field action in d-dimensional flat spacetime

$$S[\phi] = \int \mathcal{L}(\phi, \partial \phi, ..., \partial^{(\wp)} \phi, x) \ d^d x, \tag{1}$$

with the following high-gradient Lagrangian (throughout the paper we will use the notations of ref. [2])

$$\mathcal{L}^{(\wp)} = \frac{1}{2} \sum_{k=1}^{\wp} \aleph_k \left( \partial_{\mu_1 \mu_2 \dots \mu_k} \phi \right)^2 - U, \tag{2}$$

where  $\wp$  is the order of the highest derivative,  $\aleph_k$  are some constants of the dimensionality  $L(\text{ength})^{2(k-1)}$  whereas  $[\phi] = L^{1-d/2}$ . In fact, the model we begin with belongs to the class of the higher derivative theories which have a long history [3] but nevertheless below we will try to take a new look at them. It is evident that this Lagrangian satisfies with the points (i) and (iii). To demonstrate the assertion (ii) let us suppose that there are no (external) interactions that means  $U = m^2 \phi^2/2$ . Further, assuming for simplicity that  $\aleph_1(\partial_{\mu_1}\phi)^2 \ll \aleph_2(\partial_{\mu_1\mu_2}\phi)^2 \ll \dots \ll \aleph_\wp(\partial_{\mu_1\dots\mu_\wp}\phi)^2$  we preserve in Eq. (2) only the highest-derivative term to obtain

$$\tilde{\mathcal{L}}_0^{(\wp)} = \frac{1}{2} \left( \partial_{\mu_1 \dots \mu_\wp} \phi \right)^2 + \frac{(-m^2)^\wp}{2} \phi^2, \tag{3}$$

where we have fixed  $\aleph_{\wp}$  and rescaled the scalar field to be of the dimensionality  $\mathcal{L}^{\wp-d/2}$ . The equation of motion which follows,

$$\left[\Box^{\wp} - (-m^2)^{\wp}\right]\phi = 0, \tag{4}$$

contains the usual Klein-Gordon because  $\Box^{\wp} - (-m^2)^{\wp} = [\Box^{\wp-1} - (-m^2)^{\wp-1}] (\Box + m^2)$ , etc. We will call the field/particles governed by such equations as the  $\wp$ -free particles understanding that the standard notion of the free particle recovers when  $\wp \to 1$ . This is the reflection of the suggestion (ii) above. From the viewpoint of the deterministic principle it means that the world-line history of a particle becomes depending on the initial data not only of fields and their first derivatives but also of their higher derivatives that agrees with the assertion (i) and seems to be true in the highly inhomogeneous and rapidly varying Desert. The satisfaction with the correspondence principle can also be seen in the Fourier space where Eq. (4) takes the algebraic form

$$\left(k^{2\wp} - m^{2\wp}\right)\phi_k = 0. \tag{5}$$

Expanding it in series near the point  $\wp = 1$  we have

$$\frac{k^2}{\Lambda^2} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left( (\wp - 1) \ln \frac{k^2}{\Lambda^2} \right)^n \right] - \frac{m^2}{\Lambda^2} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left( (\wp - 1) \ln \frac{m^2}{\Lambda^2} \right)^n \right] = 0, \quad (6)$$

where  $\Lambda$  is some characteristic momentum. One can see that these series converge to the Fourier-image of the usual Klein-Gordon not only when  $\wp \to 1$  but also when  $k^2$  and  $m^2 \to \Lambda^2$ . Besides, to obtain, e.g., the exponent  $k^{2\wp}$  it should be  $k > \Lambda$  otherwise we would have either  $\wp < 1$  or alternating series which converge to an oscillating function. Thus, we see that the usual ( $\wp = 1$ )-field theories and models appear to be the low-energy limits of the high-gradient ones with respect to the characteristic value.

It is interesting that looking at the Fourier expansions above one can observe that the high-gradient theories also appear to be the generalization of some theories which do not have any clear local representation in coordinate space. These are, for example, fractional-derivative theories (if we take noninteger  $2\wp$ ) and logarithmic-momenta theories (if we take a finite number of terms in the series). In coordinate space the formers have doubtful (non-local) definition through integrals whereas the latters have no local derivative formulation at all because appear when considering dynamical systems with constraints in the phase space.

Further, at even  $\wp$  there arises the symmetry between virtual tachyons and real particles. The formers are known to do not satisfy with causal on-mass-shell requirement and have states with complex mass. The number of the real-virtual partners (including "antiparticles"  $m \to -m$ ) is determined in fact by zeros of left-hand side of Eq. (5): at  $\wp = 2$  we have VR fourplet  $\pm m$ ,  $\pm im$ , at  $\wp = 4$  we have VR octuplet  $\pm m$ ,  $\pm im$ ,  $\pm i\sqrt{i}m$ , etc. It is unclear whether VR symmetry is just broken

in low-energy regime (like supersymmetry) or does not exist in Nature at all. It seems that the latter is more probable because, e.g., the  $\wp$ -generalized propagator for boson fields has non-removable singularities at even  $\wp$ , as will be shown below. Be that as it may, fermionic fields do not have the VR symmetry: in the Fermi case we have, to a highest-order gradient,

$$\tilde{\mathcal{L}}^{(\wp)} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\Box^{\frac{\wp-1}{2}}\psi - U(\bar{\psi}, \ \psi), \tag{7}$$

and in the case of ( $\wp$ -)free field,  $U_0 = (-1)^{(\wp+1)/2} m^{\wp} \bar{\psi} \psi$ , we obtain the following  $\wp$ -Dirac equation

$$\left[i\gamma^{\mu}\partial_{\mu}\Box^{\frac{\wp-1}{2}} - (-1)^{\frac{\wp-1}{2}}m^{\wp}\right]\psi = 0, \tag{8}$$

which is meaningful at odd  $\wp$  only.

Noether theorem. The important step we proceed now is the formulation of the notion of conserved current which is crucial both for creating the high-gradient models with specific symmetries and for functional-integral quantization of high-gradient theories [4]. We demonstrate the Noether theorem for the high-gradient scalar field but the generalization for fields with more spacetime or internal indices is trivial. So, let us suppose that the action (1) is invariant under the group of transformations  $x'^{\mu} = x^{\mu} + \delta x^{\mu}$ ,  $\phi'(x) = \phi(x) + \delta \phi(x)$ , where the variations are characterized by the infinitesimal parameter(s)  $\delta \omega^{\mu}$ :  $\delta x^{\mu} = X^{\mu}_{\nu} \delta \omega^{\nu}$ ,  $\delta \phi(x) = \Phi_{\nu} \delta \omega^{\nu} - (\partial_{\nu} \phi) \delta x^{\nu}$ , and we are working in the standard approximation to the order  $O(\delta x^2)$ . Let us give the final result: provided the generalized equations of motion,

$$0 = \frac{\delta S}{\delta \varphi} \sim \frac{\partial \mathcal{L}}{\partial \phi} + \sum_{k=1}^{\wp} (-1)^k \partial_{\mu_1 \dots \mu_k} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu_1 \dots \mu_k} \phi)},$$

are hold, the conserved Noether  $\wp$ -current,  $\partial_{\mu}J^{\mu}_{\nu}=0$ , has the following form

$$J^{\mu}_{\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_{\mu}\phi)} \Phi_{\nu} - \theta^{\mu}_{\alpha} X^{\alpha}_{\nu} + \sum_{k=2}^{\wp} \check{\partial}_{\mu_{1} \dots \mu_{k-1}} \left[ \frac{\partial \mathcal{L}}{\partial (\partial_{\mu_{1} \dots \mu_{k-1} \mu} \phi)} \cdot (\Phi_{\nu} - X^{\alpha}_{\nu} \partial_{\alpha} \phi) \right], \quad (9)$$

where

$$heta_{
u}^{\mu} = \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi)} \partial_{
u} \phi - \delta_{
u}^{\mu} \mathcal{L},$$

and it is assumed that

$$\check{\partial}_{\mu_1..\mu_j}[A \cdot B] \equiv (\partial_{\mu_1..\mu_j} A) B - (\partial_{\mu_1..\mu_{j-1}} A) \partial_{\mu_j} B + \dots 
+ (-1)^j A \partial_{\mu_1..\mu_j} B.$$

The conserved  $\wp$ -charge can be defined in a standard way as the integral of  $J_{\nu}^{0}$  over the spatial (d-1)-volume V. It should not be forgotten that, first, the Noether current can obtain (or loose) extra indices in dependence

on indices of both fields and transformation generators, and, second, that the symmetrization over its indices can always be imposed.

Further, at the pure translations  $(X^{\mu}_{\nu} = \delta^{\mu}_{\nu}, \Phi = 0)$  we have the  $\wp$ -generalized energy-momentum tensor

$$T^{\mu}_{\nu} = \theta^{\mu}_{\nu} + \sum_{k=2}^{\wp} \check{\partial}_{\mu_{1}...\mu_{k-1}} \left[ \frac{\partial \mathcal{L}}{\partial (\partial_{\mu_{1}...\mu_{k-1}\mu} \phi)} \cdot \partial_{\nu} \phi \right], \quad (10)$$

which replaces the ordinary one in the Desert, whereas at the purely internal global U(1) transformations of a complex componentless field,  $\psi \to e^{-i\Lambda}\psi$ ,  $\psi^* \to e^{i\Lambda}\psi^*$ , the conserved current is given as a superposition of the usual one and the sum of the high-gradient currents:

$$J^{\mu} = i \left[ \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi)} \phi - \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi^{*})} \phi^{*} \right] + \sum_{k=2}^{\wp} J^{\mu}_{(k)},$$

$$J^{\mu}_{(k)} = i \, \check{\partial}_{\mu_{1} \dots \mu_{k-1}} \left[ \frac{\partial \mathcal{L}}{\partial (\partial_{\mu_{1} \dots \mu_{k-1} \mu} \psi)} \cdot \psi - \frac{\partial \mathcal{L}}{\partial (\partial_{\mu_{1} \dots \mu_{k-1} \mu} \psi^{*})} \cdot \psi^{*} \right],$$

and it is doubtless that the appearance of high-gradient currents for any given symmetry is inevitable in Desert.

Quantization. Once we have the notion of the conserved current we can quantize the theory using Schwinger's currents formulation and functional integral methods [5]. Let us begin with the  $\wp$ -Klein-Gordon equation in the presence of a source current

$$\left[\Box^{\wp} - (-m^2)^{\wp}\right]\phi_0 = J,\tag{11}$$

hence its solution is

$$\phi_0(x) = -\int \Delta_F^{(\wp)}(x - y) J(y) d^d y$$
 (12)

where  $\Delta_F^{(\wp)}(x)$  is the  $\wp$ -Feynman propagator obeying the  $\wp$ -Klein-Gordon equation with the distributional source

$$\left[\Box^{\wp} - (-m^2)^{\wp}\right] \Delta_F^{(\wp)}(x) = -\delta^{(d)}(x). \tag{13}$$

The vacuum-to-vacuum transition amplitude is given by

$$Z_0[J] = \exp\left[-\frac{i}{2} \int J(x) \, \Delta_F^{(\wp)}(x-y) \, J(y) \, d^d x d^d y\right].$$

If there exists an interaction  $\mathcal{L}^{(\wp)} = \mathcal{L}_0^{(\wp)} + \mathcal{L}_{\text{int}}^{(\wp)}$ ,  $U = -(-m^2)^{\wp} \phi^2/2 + U_{\text{int}}$  then the complete generating functional is (up to normalizing factor)

$$Z[J] = \exp\left[i \int \mathcal{L}_{\text{int}}^{(\wp)} \left(\frac{1}{i} \frac{\delta}{\delta J}\right) d^d x\right] Z_0[J], \qquad (14)$$

which should be expanded in terms of perturbation series, Feynman diagrams, and all that. The crucial point here is the  $\wp$ -Feynman propagator. Resolving Eq. (13) we

obtain (omitting the imaginary Breit-Wigner terms in the denominator)

$$\Delta_F^{(\wp)}(x) = \frac{(-1)^{\wp - 1}}{(2\pi)^d} \int \frac{e^{-ikx}}{k^{2\wp} - m^{2\wp}} d^d k, \tag{15}$$

or, alternatively, performing the Wick rotation we have in Euclidean space:

$$\Delta_F^{(\wp)}(x) = \frac{i}{(2\pi)^d} \int \frac{e^{-ik_E x}}{k_E^{2\wp} + (-1)^{\wp+1} m^{2\wp}} d^d k_E,$$

and considering Eq. (6) it is easy to see that the  $\wp$ -Feynman propagator generalizes the usual one for energies above (and, by the way, perhaps below [6]) some characteristic scale  $\Lambda$ . The fermionic  $\wp$ -Feynman propagator has a similar form ( $\wp$  must be odd):

$$\Delta_F^{(\wp)}(x) = \frac{(-1)^{\frac{\wp-1}{2}}}{(2\pi)^d} \int \frac{e^{-ipx}}{pp^{\wp-1} - m^\wp} d^d p, \tag{16}$$

following the  $\wp$ -Dirac equation above.

Unified all-in-one mass states and origin of generations. Let us consider first the bosonic case. If instead of the simplified theory (3) one considers the more general one (2) then it removes (completely or particularly) the degeneration of the propagator's pole:

$$\frac{1}{k^{2\wp} - m^{2\wp}} \to \frac{1}{(k^2 - m_1^2)^{a_1} \dots (k^2 - m_N^2)^{a_{N\wp}}}, \tag{17}$$

where  $a_i$   $(i = 1, 2, ..., N_{\wp})$  are the multiplicities of roots,  $1 \leq N_{\wp} \leq \wp$  becomes the number of multiplemass states,  $m_i = m_i(\aleph_i, m)$  are masses of states. Thus, in principle one particle degree of freedom can have several 1  $\leq$   $N_{\wp}$   $\leq$   $\wp$  mass states which will be therefore named as the unified "all-in-one" multi-mass states. It is interesting phenomenon and perhaps it can be the natural explanation of such mysteries as the origin of masses and the existence of several generations of the particles (as well as oscillations between them) which have very similar characteristics (charge, spin, etc.) but sharply different values of masses. At least, it explanates why the maximal number of fermionic generations is an odd number. The maximally possible number of generations is determined by that of the fermionic  $\wp$ -Feynmann propagator's poles (fully non-degenerate case), therefore, the more we deepen into the Desert the bigger are  $\wp$ 's and hence the number of generations. One may object that the fermionic propagator in SM has only one pole whereas the number of generations is three. However, it should not be forgotten that SM is infinite in ultraviolet domain hence incomplete [1], therefore, it "describes" rather than "explanates" the existing symmetries and generations. This at least means that SM should be updated in its highest-energy scale margin to take into account highgradient corrections, see the program-minimum below.

However the most striking benefit we have obtained is the absolute power over the divergences. Returning to Eq. (15) we see that  $\wp$ -Feynman propagators converge for  $\wp > d/2$  (bosons) and  $\wp > d$  (fermions). In fact, it is the consequence of the fact that high-gradient theories properly take into account high-energy corrections. It immediately means that all the integrals of the theory, including the self-energy loops, have good (or, at least, controlled) ultra-violet behavior because they are given as the products of propagators

$$I \sim \int \Delta_F^{(\wp)}(x_1 - y_1) ... \Delta_F^{(\wp)}(y_k - y_M) d^d y_1 ... d^d y_M,$$

in dependence on the topology of a concrete diagram. It should be noted that N-supersymmetry-based models have much less well-defined ultraviolet behaviour in the most realistic case N=1 besides there exists the problem of superpartners. Besides, the self-energy loops  $\Delta_F^{(\wp)}(0)$  become finite but, unlike N=1 rigid supersymmetry theories, not necessary zero and hence can be regarded as the fundamental characteristics of vacuum hence of the Nature.

Symmetry. Any concrete model within the frameworks of a given theory can be constructed founding on a particular group of symmetry. Even if high-gradient and ordinary models have the same symmetry their properties (first of all, currents and charges) in the general case  $\wp \not\to 1$  are different due to appearance of the highgradient currents (however, it does not mean yet that the conservation law of, e.g., electrical charge breaks in highgradient theories: simply the definition of a charge should be updated to include high-gradient corrections). Therefore, when trying to construct the consistent beyond-SM physics it is necessary to understand which symmetry survives in the Desert. Generally speaking, there we can encounter the fact that many habitual gauge or even global and discrete symmetries (such as CPT) become approximate. On the other hand, there can appear a number of explicit and hidden symmetries which are inherent to high-derivative systems [7] including the high-derivative supersymmetries [8] (with the known simplifications caused by nature of Grassmanian variables) and Riemann-Weyl-Cartan ones (in the vicinity of the high-energy border of the Desert where spacetime cannot be supposed flat or even locally flat). All these circumstances can sufficiently complicate the constructing of physical models but nobody asserts that trips across deserts are easy. Besides, it should be remembered that we have the large advantage of possessing the well-defined finite perturbation theory, therefore, highgradient models are calculable hence verifiable independently of whether they are "renormalizable" or no.

Further, let us demonstrate the two viewpoints concerning ways of the heuristic constructing of desert models illustrating them on the simple d-dimensional example, quantum electrodynamics (which, however, should be regarded just as a toy illustration rather than realistic extrapolation because SM contains QED only as a part of the underlying non-Abelian theory):

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + m\bar{\psi}\psi + \mathcal{L}_{F},\tag{18}$$

where  $D_{\mu} = \partial_{\mu} - ieA_{\mu}$ , and  $\mathcal{L}_F = (1/4)F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{GF}$ , where  $\mathcal{L}_{GF}$  is the gauge-fixing term. The minimal  $\wp$ -generalization of the theory leads to the redefinition of covariant derivative such that

$$\mathcal{L}_{\wp QED} = i\bar{\psi}\gamma^{\mu}D_{\mu}^{(\wp)}\psi + (-1)^{\frac{\wp-1}{2}}m^{\wp}\bar{\psi}\psi + \aleph\mathcal{L}_F, \quad (19)$$

where  $D_{\mu}^{(\wp)}=\partial_{\mu}\Box^{\frac{\wp-1}{2}}-ieA_{\mu}$ ,  $\aleph$  is some constant of the dimensionality  $\mathcal{L}^{2(\wp-1)}$ , and  $[\psi]=\mathcal{L}^{(\wp-d)/2}$ ,  $[A_{\mu}]=$  $L^{2-\wp-d/2}$ . One can see that the  $D^{(\wp)}$ -term preserves the global U(1) symmetry but explicitly breaks the gauge one. Then the first viewpoint tells the following: such a gauge symmetry does survive in the Desert and hence additional terms should be inserted in the Lagrangian to restore it that can easily be done. The previous definitions of currents and charges are only approximate, in high-energy regime they should be replaced with the  $\wp$ generalized ones. Unlike the first point of view the second one asserts that: (a) this symmetry eventually dies in the Desert, (b) this Lagrangian satisfies with the correspondence principle, therefore, it can be considered "as is". and hence the main questions are where it goes, what are its symmetries, conserved values, etc. Of course, the represented opposite views are quite radical, and it is clear that the correct way should lay somewhere between them.

Program-minimum. All the above-mentioned recipes of the consistent constructing of the Desert field theory give in aggregate the program-maximum. It is necessarv also to outline the minimal or "regularization without renormalization" program which is applicable in the lower-bound margin of the Desert and less painful because it can be applied to the Standard Model in the present form without the full  $\wp$ -generalization of matter (fermionic) or gauge fields. Let us recall the set of SM particles: (i) gauge bosons  $W^{\pm}$ , Z, A which are expressed as superposition of SU(2) gauge triplet  $W^a$  and U(1) field B, (ii) fermions: three generations of leptons and quarks, (iii) eight gluons  $G^a_\mu$ , (iv) Higgs doublet, (v) Faddeev-Popov ghosts (unphysical scalar particles with the Fermi-Dirac statistics which were introduced to preserve unitarity [9]): eight gluon's ones and four ghosts of the gauge bosons. The ghosts are of special interest now, see also ref. [10]. Due to their unphysical properties they can appear only as internal lines of diagrams and play the role of auxiliary particles. Therefore, the program-minimum suggests the following: if input-output particles (external lines of Feynman graphs) have energies comparable with the SM scale it is enough to  $\wp$ -generalize only the internal-line particles in such a way that the Standard Model would become finite without UV cutoff, see [11] and references therein.

The final question we mention now is the one about the concrete value of  $\wp$ . The most plausible way would be to declare  $\wp$  as an auxiliary field and to minimize action with respect to it. Unfortunately, nobody knows how to variate the derivative order (here we are not considering the tricks such as transition to the Fourier space where theory becomes defined even for non-integer  $\wp$  [6]) so the tentative recipe is to conduct calculations assuming  $\wp$  as general as possible, and then in final expressions to choose the value either by involving theoretical considerations such as minimization of energies, preserving of fundamental symmetries, etc., or directly comparing with experimental data.

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